

In section 2 of this paper, we will describe the development of a first-year mathematics subject for a biomedicine degree programme at a research-intensive Australian university. The subject teaches mathematical concepts and modelling skills which are relevant to biomedical science, drawing on key mathematical models from biomedical research. In section 3, we present some findings from a statistical analysis of student grades over several semesters. This gives some insight into the importance of mathematical background, mathematical achievement, biological background and gender as predictors for achievement in biomedical mathematics, and complements previous research in the context of more generalist first-year mathematics subjects.

2 A mathematical modelling subject for undergraduate biomedicine

2.1 Context

The Bachelor of Biomedicine programme at the University of Melbourne is a 3-year undergraduate degree course. The programme offers a range of specialisations, such as immunology, genetics, and bioengineering systems. Many of the students in the programme aspire to continue to postgraduate medicine or other professional health qualifications, though the programme also offers pathways into graduate study in other fields such as engineering. In many cases, entry to these postgraduate courses requires a minimum grade point average (GPA) from the undergraduate degree. The Bachelor of Biomedicine has been offered at The University of Melbourne since 2008, following a major overhaul of the University's degree programmes. The intake is approximately 400-500 students each year.

In addition to a competitive selection process based primarily (for domestic students at least) on a ranking of all completing secondary school students, students seeking entry to the Biomedicine programme must also satisfy specific prerequisite requirements, usually by completing particular subjects in their final years of school. From 2008-2010 the prerequisite requirements included studies in English, chemistry and at least one of biology, physics, intermediate level mathematics (which includes calculus) or higher level mathematics. One consequence of this was that students entering the programme during this period had a wide range of mathematics backgrounds, ranging from advanced mathematics down to year 10 mathematics only. From 2011 onwards the prerequisite requirements were changed to English, chemistry, and intermediate or higher level mathematics; hence all students entering the programme from 2011 onwards had studied mathematics to at least intermediate level.

Since its inception the Bachelor of Biomedicine programme has included core first-year studies from several science disciplines, including biology, chemistry, mathematics, physics and statistics. In the early years of the programme, the possibility of a mathematics subject which would teach mathematics in biomedical contexts was discussed, however it was considered infeasible to develop such a subject due to the wide range of mathematics backgrounds of students at this time. Students were instead required to complete one of a selection of mainstream first-year calculus-based mathematics subjects, with the choice of subject based on the level of mathematics studied previously. Students were also required to complete a separate introductory statistics subject. With the introduction of a mathematics prerequisite for entry to the degree in 2011, the diversity of mathematics background was decreased, and the possibility of a specialised mathematics subject was revisited.

In 2013 a new undergraduate mathematics subject "*Mathematics for Biomedicine*" was developed for the Bachelor of Biomedicine programme. Most students were required to complete this subject², along with other core subjects including introductory statistics, chemistry, physics and biology, as part of their degree programme. The subject *Mathematics for Biomedicine* was

² Students intending to complete a major in bioengineering systems (a potential pathway to postgraduate engineering courses) would complete a sequence of standard calculus and linear algebra subjects instead.

developed by the authors over the summer of 2013, with the first author responsible for curriculum design and the second author responsible for development of supporting resources. We now describe the subject and some factors informing its design.

2.2 Subject design requirements

The main aim of the subject was to teach mathematics that is *relevant* to biomedical science, using *authentic* mathematical models from biomedicine. The intention was to show students the value of quantitative modelling in biomedical science, for instance as a tool to help understand natural phenomena. Further aims were to complement other Bachelor of Biomedicine subjects where possible, as well as to develop generic skills such as problem-solving, collaboration and mathematical communication. The development of procedural fluency, a common goal of many generic calculus subjects, was a lesser concern, though students were nevertheless expected to gain fluency in several techniques from the subject. The subject design was developed in consultation between staff from Mathematics and Statistics and from Biomedicine, and in view of previous work such as (Matthews et al., 2010) and (Poladian, 2013).

Further considerations arise from the nature of the student cohort. All students in the cohort have completed a minimum of intermediate level school mathematics, which includes manipulation and graphing of functions, basic differential and integral calculus, probability and statistics. Approximately half of the students have also taken additional school calculus, and some have also taken further non-calculus mathematics subjects. Many of the students aspire to enter postgraduate medicine courses for which entry is competitive and based partly on grade point averages; hence the student cohort is very competitive and sensitive to anything that could be seen to advantage some students. Thus it was required to make the subject content accessible to students with an intermediate mathematics background while ensuring a 'level playing-field' for students with or without additional mathematics. Finally, as a standalone unit which does not serve as prerequisite for any later subjects, there was no requirement to cover specific content for later subjects.

2.3 The subject

Content

Three biological contexts were chosen around which to structure the subject: population genetics, reaction kinetics/systems biology, and infectious disease modelling. The subject content is divided into three topics corresponding to these three contexts. Each topic begins with a discussion of the biological context and several motivating questions: for example, "how will the relative frequencies of a gene change in a population over time?", "will a disease cause an epidemic in a particular population, and if so, how could we prevent it?". The mathematical models and techniques studied in the subject arise naturally as we seek to answer these questions.

Over the course of the semester, several recurring themes are seen to emerge in many of the different biological and mathematical contexts investigated in the subject. These include concepts such as equilibrium and stability, long-term and limiting behaviour, deterministic vs. stochastic models, and the use of graphical techniques to gain insight into the qualitative aspects of a model or solution. Links also emerge between mathematical concepts and models studied in different biological contexts, illustrating the ubiquity of certain mathematical ideas in science. A summary of the mathematical content covered in each of the topics, and some of the recurring themes, is given in Table 1.

Topic	Mathematical techniques & concepts	Recurring themes
Population genetics (5 weeks)	Linear and non-linear first-order difference equations (aka recurrence relations) Absorbing Markov chains	Equilibrium Stability Long-term behaviour Solving vs verifying Homogeneous and particular solutions Graphical techniques Deterministic vs stochastic
Reaction kinetics/ systems biology (4 weeks)	First order linear and non-linear ordinary differential equations (ODEs) Numerical methods Systems of ODEs Phase plane analysis Quasi-steady-state approximations	
Infectious disease modelling (3 weeks)	Systems of ODEs Phase plane analysis	

Table 1: Biological contexts, corresponding mathematical concepts, and common themes of *Mathematics for Biomedicine*. The time spent on each topic (out of a 12 week semester) is given in parentheses.

The mathematical techniques and models used in the subject were all chosen to be accessible to students with a background of intermediate mathematics. In order to reduce any perceived advantage to students with additional prior mathematics experience, much of the mathematical content of the subject was chosen to be disjoint from that of typical advanced school mathematics courses. Concepts such as difference equations and phase planes are not typically covered in school mathematics at any level; other concepts are treated in a different way to the standard school approach. For example, instead of using the common separation of variables approach for solving simple first-order ODEs, we use the ‘principle of superposition’ to solve constant-coefficient linear ODEs by finding and combining a homogeneous and a particular solution. In many cases the focus is more on gaining a qualitative understanding of a system or model, rather than finding an exact algebraic solution. This approach is similar to that described in (Poladian, 2013). Some models used in the subject, such as Hardy-Weinberg equilibrium and Michaelis-Menten enzyme kinetics, were chosen to complement the content of other core subjects in the Bachelor of Biomedicine.

Learning activities

The learning activities used in the subject include lectures, tutorials, weekly assignments and additional practice exercises. The lectures make use of partially completed lecture slides which are supplied to students in advance. Gaps are left in the lecture slides for worked examples, derivations, diagrams, etc., which are completed during the lecture using a document camera or tablet PC. The tutorials run as active ‘whiteboard’ tutorials, where students work collaboratively through tutorial exercises in groups of 3-4 on whiteboards around the room. This style of tutorial offers active, peer learning in an engaging format. The use of partially completed lecture slides and whiteboard tutorials is standard in most first and second-year subjects offered by our School. For further discussion of the merits and history of the whiteboard tutorial style see (Seaton, King, & Sandison, 2014). Tutorial questions include routine mathematical exercises, applications of particular mathematical models, and discussion of modelling issues. The tutorials are highly valued by students, and have consistently been one of the most commonly mentioned aspects of the subject in response to the question “What were the best aspects of this subject?” on the end-of-semester Subject Experience Survey. This is consistent with other subjects from our school which also use whiteboard tutorials. Students are also given short weekly assignments as formative assessment as well as additional non-assessed practice exercises.

